The Hamilton Operator and Quantum Vacuum for Nonconformal Scalar Fields in the Homogeneous and Isotropic Space

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Abstract. The diagonalization of the metrical and canonical Hamilton operators of a scalar field with an arbitrary coupling, with a curvature in N-dimensional homogeneous isotropic space is considered in this paper. The energy spectrum of the corresponding quasiparticles is obtained and then the modified energy-momentum tensor is constructed; the latter coincides with the metrical energy-momentum tensor for conformal scalar field. Under the diagonalization of corresponding Hamilton operator the energies of relevant particles of a nonconformal field are equal to the oscillator frequencies, and the density of such particles created in a nonstationary metric is finite. It is shown that the modified Hamilton operator can be constructed as a canonical Hamilton operator under the special choice of variables.

1 Introduction

Our aim in this paper is the investigation of the Hamiltonian diagonalization method and the definition of the Hamilton operator and quantum vacuum of nonconformal scalar field in nonstationary homogeneous isotropic space. Quantum field theory in curved space-time (see monographs [1, 2]) has important applications to cosmology and astrophysics. However there are several problems that have not been finally solved until the present time. One of them is the definition of vacuum state and the notion of elementary particle in curved space-time; this is due to the absence of the group of symmetries such as the Poincare group in Minkowsky space. This problem for nonconformal scalar field is under active discussion even in the case of homogeneous isotropic space [3, 4, 5]. As a consequence of various definitions of vacuum states we have a variety of calculated quantum characteristics of nonconformal scalar fields in curved space.

In [6, 7] it was shown that in the case of arbitrary coupling of scalar fields with curvature additional, nonconformal contributions are dominant in vacuum averages of the energy-momentum tensor. It should be also mentioned that the investigation of nonconformal scalar fields is not only of independent interest;

this investigation is caused by impossibility of preservation of conformal invariation of effective action and the usual action in the case of interacting quantized field [2].

In the definition of vacuum and in the formulation of the problem of particles creation in curved space-time two known approaches are widely used: the Hamiltonian diagonalization procedure [1] offered in [8, 9], and the so-called "adiabatic" procedure [2] offered in [10]. Supposing that a quantum of energy corresponds to a particle, then observation of particles at some moment (according to quantum mechanics) means to find the Hamilton operator's eigenstate; this is taken into account automatically in the diagonalization approach. In nonstationary metrics the diagonalization procedure is realized by the time-dependent Bogolyubov transformations (see below). If the operators of these transformations are Hilbert-Schmidt operators then the representations of commutation relations are unitarily equivalent for both the old and the new creation and annihilation operators [11]. However, the use of the Hamilton operator constructed from the metrical energy-momentum tensor, successful in the conformal case [1], leads to the difficulties related to an infinite density of created particles in the nonconformal case [12]. At the same time essential problems and ambiguities take place in adiabatic approach also [5].

In this paper we will consider the complex scalar field with arbitrary coupling, with curvature in N-dimensional homogeneous isotropic space. In section 2 all necessary information is given and nonconformal scalar field quantizing in N-dimensional homogeneous isotropic space is defined. In section 3 the metrical and canonical Hamilton operators diagonalization is carried out, and the energies of corresponding quasiparticles are calculated, and conditions connected with demand of the Hamilton operators diagonality are investigated. In section 4 the modified energy-momentum tensor is defined so that the quasiparticles from diagonalization of corresponding Hamilton operator have energies coinciding with the oscillator frequency of the wave equation. It is shown that such Hamiltonian can be defined as canonical under a certain choice of canonical variables. It is proved that the density of particles being created in a nonstationary metric is finite and the results of given investigations are summarize.

The system of units in which the Planck constant ($\hbar$) and light velocity are equal 1 is used in the paper.

2 Quantizing of scalar field in homogeneous isotropic space

We consider the complex scalar field $\phi(x)$ of mass m satisfying the equation

$$(\nabla_i \nabla^i + \xi R + m^2)\, \phi(x) = 0\,, \tag{1}$$

where ∇_i is covariant differentiation, R is the scalar curvature, $x = (t, \mathbf{x})$, ξ is the coupling constant. The value $\xi = \xi_c = (N-2)/\,[\,4\,(N-1)]$ corresponds to conformal coupling in space-time of dimension N ($\xi_c = 1/6$ if $N = 4$). The

equation (1) is conformaly invariant if $\xi = \xi_c$ and $m = 0$; the value $\xi = 0$ reduces to the case of minimal coupling.

The metric of N-dimensional homogeneous isotropic space-time is

$$ds^2 = g_{ik} dx^i\, dx^k = dt^2 - a^2(t)\, dl^2 = a^2(\eta)\, (d\eta^2 - dl^2)\,, \tag{2}$$

where $dl^2 = \gamma_{\alpha\beta}\, dx^\alpha\, dx^\beta$ is the metric of $(N-1)$-dimensional space with the constant curvature $K = 0, \pm 1$.

The equation (1) can be obtain by varying the action with Lagrangian density

$$L(x) = \sqrt{|g|}\ [\, g^{ik} \partial_i \phi^* \partial_k \phi - (m^2 + \xi R) \phi^* \phi\,]\,, \tag{3}$$

where $g = \det(g_{ik})$.

The canonical energy-momentum tensor of the scalar field is

$$T_{ik}^{can} = \partial_i \phi^* \partial_k \phi + \partial_k \phi^* \partial_i \phi - g_{ik} |g|^{-1/2} L(x)\,. \tag{4}$$

The metrical energy-momentum tensor which can be obtained by varying the action of g_{ik} has a form [13]:

$$T_{ik} = T_{ik}^{can} - 2\,\xi\, [R_{ik} + \nabla_i \nabla_k - g_{ik} \nabla_j \nabla^j]\, \phi^* \phi\,, \tag{5}$$

where R_{ik} is Ricci tensor.

In the metric (2) the equation (1) takes the form

$$\phi'' + (N-2) \left(\frac{a'}{a} \right) \phi' - \Delta_{N-1}\, \phi + (m^2 + \xi R)\, a^2 \phi = 0\,, \tag{6}$$

where Δ_{N-1} is the Laplace-Beltrami operator in $(N-1)$-dimensional space, and the prime denotes the derivative with conformal time η.

For the function $\tilde{\phi} = a^{(N-2)/\,2} \phi$ the equation (6) takes the form without the first derivative in time

$$\tilde{\phi}'' - \Delta_{N-1}\, \tilde{\phi} + \left(m^2 a^2 - \Delta\xi a^2 R + ((N-2)/\,2)^2\, K \right)\, \tilde{\phi} = 0\,, \tag{7}$$

where $\Delta\xi = \xi_c - \xi$. The variables in the equations (6), (7) can be separated; namely, for $\tilde{\phi} = g_\lambda(\eta) \Phi_J(\mathbf{x})$ we have

$$g_\lambda''(\eta) + \Omega^2(\eta)\, g_\lambda(\eta) = 0\,, \tag{8}$$

and

$$\Delta_{N-1}\, \Phi_J = -(\lambda^2 - ((N-2)/\,2)^2\, K)\, \Phi_J\,; \tag{9}$$

$\Omega(\eta)$ is the oscillator frequency

$$\Omega^2(\eta) = m^2 a^2 + \lambda^2 - \Delta\xi\, a^2 R\,, \tag{10}$$

J is a set of indexes (quantum numbers) numbering the eigenfunctions of the Laplace-Beltrami operator. It should be noted that the eigenvalues of the operator $-\Delta_{N-1}$ are not negative and we have the inequality

$$\lambda^2 - ((N-2)/\,2)^2\, K \geq 0\,.$$

For quantization we decompose the field $\tilde{\phi}(x)$ by the complete set of the solutions of (7), i.e.

$$\tilde{\phi}(x) = \int d\mu(J) \left[\tilde{\phi}^{(-)}_{\bar{J}}\, a^{(-)}_{\bar{J}} + \tilde{\phi}^{(+)}_{J}\, a^{(+)}_{J} \right] ; \tag{11}$$

here $d\mu(J)$ is the measure in the space of the Laplace-Beltrami Δ_{N-1} eigenvalues

$$\tilde{\phi}^{(+)}_{J}(x) = \frac{1}{\sqrt{2}}\, g_\lambda(\eta)\, \Phi^*_J(\mathbf{x}) \ , \qquad \tilde{\phi}^{(-)}_{\bar{J}}(x) = \frac{1}{\sqrt{2}}\, g^*_\lambda(\eta)\, \Phi_{\bar{J}}(\mathbf{x}) \ , \tag{12}$$

$\Phi_J(\mathbf{x})$ is orthonormal eigenfunctions of Δ_{N-1} operator, and $\bar{J}$ is a set of quantum numbers of the function complex conjugated to the function Φ_J. In $(N-1)$-dimensional spherical coordinates for $J = \{\lambda\,, l\,, \ldots\,, m\}$ we have $\bar{J} = \{\lambda\,, l\,, \ldots\,, -m\}$.

Substituting expansion (11) in the expression for conserved charge

$$Q = i \int\limits_{\Sigma} \left(\tilde{\phi}^* \partial_0 \tilde{\phi} - (\partial_0 \tilde{\phi}^*)\, \tilde{\phi} \right) \sqrt{\gamma}\, d^{N-1}x \, , \tag{13}$$

where $\gamma = \det(\gamma_{\alpha\beta})$, Σ is a space-like hypersurface $\eta=$const, and imposing the normalization condition

$$g_\lambda\, g^{*\prime}_\lambda - g'_\lambda\, g^*_\lambda = -2i \ , \tag{14}$$

we obtain

$$Q = \int d\mu(J)\, \left(\overset{*}{a}{}^{(+)}_J a^{(-)}_J - \overset{*}{a}{}^{(-)}_{\bar{J}} a^{(+)}_{\bar{J}} \right) \ . \tag{15}$$

The metrical Hamiltonian is expressed in terms of the metrical energy-momentum tensor (5) by [1]:

$$\begin{aligned} H(\eta) &= \int\limits_{\Sigma} \zeta^i\, T_{ik}(x)\, d\sigma^k = \int\limits_{\eta=\text{const}} \zeta^0\, T_{00}(x)\, g^{00} \sqrt{|g|}\ d^{N-1}x \quad = \\ &= a^{N-2}(\eta) \int\limits_{\eta=\text{const}} T_{00}(x) \sqrt{\gamma}\ d^{N-1}x \, , \end{aligned} \tag{16}$$

where $(\zeta^i) = (1, 0, \ldots, 0)$ is the time-like conformal Killing vector.

The quantization is realized by the commutation relations

$$\left[a^{(-)}_J,\ \overset{*}{a}{}^{(+)}_{J'} \right] = \left[\overset{*}{a}{}^{(-)}_J,\ a^{(+)}_{J'} \right] = \delta_{JJ'} \ , \quad \left[a^{(\pm)}_J,\ a^{(\pm)}_{J'} \right] = \left[\overset{*}{a}{}^{(\pm)}_J,\ \overset{*}{a}{}^{(\pm)}_{J'} \right] = 0 \, . \tag{17}$$

The Hamilton operator (16) can be written, through the $a^{(\pm)}_J$, $\overset{*}{a}{}^{(\pm)}_J$ operators in the form

$$\begin{aligned} H(\eta) \quad &= \quad \int d\mu(J) \Big\{ E_J(\eta)\, \left(\overset{*}{a}{}^{(+)}_J a^{(-)}_J + \overset{*}{a}{}^{(-)}_{\bar{J}} a^{(+)}_{\bar{J}} \right) + \\ &+ \quad F_J(\eta)\ \overset{*}{a}{}^{(+)}_J a^{(+)}_{\bar{J}} + F^*_J(\eta)\ \overset{*}{a}{}^{(-)}_{\bar{J}} a^{(-)}_J \Big\} \, , \end{aligned} \tag{18}$$

where

$$E_J(\eta) = \frac{1}{2}\left\{ |g'_\lambda|^2 + D_\lambda(\eta)\,|g_\lambda|^2 - Q(\eta)\,(|g_\lambda|^2)' \right\} , \tag{19}$$

$$F_J(\eta) = \frac{(-1)^m}{2}\left\{ {g'_\lambda}^2 + D_\lambda(\eta)\,g_\lambda^2 - Q(\eta)\,(g_\lambda^2)' \right\} , \tag{20}$$

$$D_\lambda(\eta) = m^2a^2 + \lambda^2 + \Delta\xi\,(N-1)\,(N-2)\,(c^2 - K)\,, \tag{21}$$

$$Q(\eta) = \Delta\xi\,2\,(N-1)\,c\,, \tag{22}$$

and $c = a'(\eta)/a(\eta)$. The Hamilton operator corresponding to canonical energy-momentum tensor (4) has the form (18) with (21) and (22) replaced by

$$\begin{aligned} D_\lambda(\eta) \quad &= \quad m^2a^2 + \lambda^2 + (N-1)\,(N-2)\,\big((\xi+\xi_c)\,c^2 - \Delta\xi\,K \big) + \\ &\qquad + 2\,\xi\,(N-1)\,c'\,, \end{aligned} \tag{23}$$

$$Q(\eta) = c\,(N-2)/\,2\,. \tag{24}$$

3 The diagonalization of the metrical and canonical Hamilton operators

The Hamilton operator (18) is diagonal at time moment η_0 in the operators $\overset{*}{a}{}^{(\pm)}_J$, $a^{(\pm)}_J$, which in this case are the creation and annihilation operators of particles and antiparticles under $F_J(\eta_0) = 0$. Utilizing (20) − (24) it may be shown that this condition is consistent with normalization (14) only when $p^2_\lambda(\eta_0) > 0$, where for the metrical Hamilton operator

$$p_\lambda(\eta) = \sqrt{m^2a^2(\eta) + \lambda^2 + 4\Delta\xi\,(N-1)^2\,(\xi\,c^2 - \xi_c\,K\,)} \tag{25}$$

and for the canonical Hamilton operator

$$p_\lambda(\eta) = p_{can,\lambda}(\eta) = \sqrt{(m^2 + \xi\,R)\,a^2(\eta) + \lambda^2 - ((N-2)/\,2)^2\,K}\,. \tag{26}$$

The requirement for the metrical Hamilton operator to be diagonal at the instant η_0, i.e., $F_J(\eta_0) = 0$, and the normalization condition lead to the initial conditions on the functions $g_\lambda(\eta_0)$:

$$g'_\lambda(\eta_0) = (2\Delta\xi(N-1)\,c + ip_\lambda(\eta_0))\,g_\lambda(\eta_0)\,, \quad |g_\lambda(\eta_0)| = 1/\sqrt{p_\lambda(\eta_0)}\,. \tag{27}$$

For the canonical Hamilton operator the initial conditions are

$$g'_\lambda(\eta_0) = ((N-2)\,c/\,2 + i\,p_{can,\lambda}(\eta_0))\,g_\lambda(\eta_0)\,, \quad |g_\lambda(\eta_0)| = 1/\sqrt{p_{can,\lambda}(\eta_0)}\,. \tag{28}$$

The state of vacuum $|\,0\!>$ corresponding to (27), (28) is defined in the standard form

$$a^{(-)}_J\,|\,0\!> = \overset{*}{a}{}^{(-)}_J\,|\,0\!> = 0\,. \tag{29}$$

For arbitrary time moment η we diagonalize the Hamilton operator in terms of $\overset{*}{b}{}^{(\pm)}_J,\ b^{(\pm)}_J$ operators which connected with $\overset{*}{a}{}^{(\pm)}_J,\ a^{(\pm)}_J$, by the time-dependent Bogolyubov transformations:

$$\left\{ \begin{array}{l} a^{(-)}_J = \alpha^*_J(\eta)\, b^{(-)}_J(\eta) - (-1)^m \beta_J(\eta)\, b^{(+)}_{\bar{J}}(\eta)\,, \\ \overset{*}{a}{}^{(-)}_J = \alpha^*_J(\eta)\ \overset{*}{b}{}^{(-)}_J(\eta) - (-1)^m \beta_J(\eta)\ \overset{*}{b}{}^{(+)}_{\bar{J}}(\eta)\,, \end{array} \right. \tag{30}$$

where $\alpha_J(\eta),\ \beta_J(\eta)$ are the functions satisfying the initial conditions $|\alpha_J(\eta_0)| = 1,\ \beta_J(\eta_0) = 0$ and the identity

$$|\alpha_J(\eta)|^2 - |\beta_J(\eta)|^2 = 1\,. \tag{31}$$

(In homogeneous and isotropic space $\alpha_J = \alpha_\lambda,\ \beta_J = \beta_\lambda$ [1]).

The substitution of the decomposition (30) in (18) gives, if we demand coefficients before nondiagonal terms $\overset{*}{b}{}^{(\pm)}_J b^{(\pm)}_J$ equal to 0, the equation

$$2(-1)^{m+1}\alpha_J\beta_J E_J + F_J\alpha_J^2 + F^*_J\beta_J^2 = 0\,. \tag{32}$$

It can be shown that the condition (32) is consistent with the normalization (14), only if $p^2_\lambda(\eta) > 0$. In that case

$$|\beta_J|^2 = E_J/\,(2p_\lambda) - 1/2 = |F_J|^2/\,(2p_\lambda\,(E_J + p_\lambda))\,. \tag{33}$$

In obtaining (33) we take into account, the result that can be checked,

$$E_J^2 - |F_J|^2 = p^2_\lambda(\eta)\cdot\left[-(g_\lambda\, g^{*\prime}_\lambda - g'_\lambda\, g^*_\lambda\,)^2/\,4\,\right]\,. \tag{34}$$

(The multiplier in square brackets equals 1 under the normalization condition (14)).

In the case of (32) and $p^2_\lambda(\eta) > 0$, the Hamilton operator (18) takes the form

$$H(\eta) = \int d\mu(J)\, p_\lambda(\eta)\, \left(\overset{*}{b}{}^{(+)}_J b^{(-)}_J + \overset{*}{b}{}^{(-)}_{\bar{J}} b^{(+)}_{\bar{J}}\right)\,. \tag{35}$$

So $p_\lambda(\eta)$ has the meaning of energy of quasiparticles corresponding to the diagonal form of the metrical Hamilton operator (and $p_{can,\lambda}(\eta)$ for the canonical Hamilton operator). For the 4-dimensional space-time the equation (25) corresponds to energy values obtained in [3] and [14].

The quasiparticle energy $p_\lambda(\eta)$ differs from the oscillator frequency $\Omega(\eta)$ of the wave equation for nonconformal field, and this leads to a series of difficulties. Thus the conditions $p^2_\lambda(\eta) > 0$ and $\Omega^2(\eta) > 0$ may be in contradiction for a nonconformal field in some cases. For example, in the case of quasi-Euclidean space ($K = 0$) and zero-mass field the condition $p^2_\lambda(\eta) > 0$ (with arbitrary λ) for the metrical case reduces to $\xi \in [\,0,\, \xi_c\,]$; but if $\xi < \xi_c\,,\ m = 0$ and $R > 0$ for low λ then we have $\Omega^2(\eta) < 0$.

It should be noted that for $p^2_\lambda(\eta) < 0$ the condition of diagonalization reduces to the vanishing of norm, energy and charge of the state with $\phi(x) \neq 0$, and this situation does not have any physical foundation.

The vacuum state defined by the equations

$$b_J^{(-)}|\,0_\eta > = \overset{*}{b}{}_J^{(-)}|\,0_\eta > = 0\,, \tag{36}$$

depends on time in the nonstationary metric. Under the initial conditions (27), (28) we have $b_J^{(\pm)}(\eta_0) = a_J^{(\pm)}$ and $|\,0_{\eta_0} >= |\,0>$. In the Heisenberg representation, the state $|\,0>$, which is vacuum at the instant η_0, is no longer a vacuum for $\eta \neq \eta_0$. It contains $|\beta_J(\eta)|^2$ quasiparticle pairs corresponding to the operators $\overset{*}{b}{}_J^{(\pm)}, b_J^{(\pm)}$ in every mode [1]. The number of the created pairs of quasiparticles in the unit of space volume (for $N = 4$) is [1]

$$n(\eta) = \frac{1}{2\pi^2 a^3(\eta)} \int d\mu(J)\, |\beta_\lambda(\eta)|^2\,. \tag{37}$$

For asymptotic solutions of equation (8) (see [15]), normalized according to (14), we can obtain from (19)—(24) that $E_J \sim \lambda$ and for nonstationary metrics $|F_J(\eta)| \sim |Q(\eta)|$ in $\lambda \to \infty$. Therefore, according to (33), this is corrected with the substitution of $p_\lambda \to p_{can,\lambda}$, and we have $|\beta_\lambda|^2 \sim \lambda^{-2}$. Consequently, the density of created quasiparticles, proportional to the integral in (37), is infinite.

So, in the diagonalization procedure, both for the metrical and the canonical Hamilton operators in nonconformal scalar fields, there is a problem of infinite density of quasiparticles created in the nonstationary metrics. In both cases the energies of corresponding quasiparticles differ from the oscillator frequency of the wave equation. It is shown below that these difficulties are absent in the case of the Hamilton operator corresponding to the modified energy-momentum tensor.

4 Modified energy-momentum tensor and modified Hamilton operator

Let us consider the modified energy-momentum tensor

$$T_{ik}^{mod} = T_{ik}^{can} - 2\xi_c\,[R_{ik} + \nabla_i\nabla_k - g_{ik}\nabla_j\nabla^j]\,\phi^*\phi\,. \tag{38}$$

From the definition (38) it is clear that for conformal scalar fields (i.e. if $\xi = \xi_c$) T_{ik}^{mod} coincides with the metrical energy-momentum tensor (5). The structure of the Hamiltonian constructed by T_{ik}^{mod} similarly to (16) is

$$\begin{aligned} H^{mod}(\eta) \;&=\; \int h(x)\, d^{N-1}x = \int d^{N-1}x\,\sqrt{\gamma}\left\{\tilde{\phi}^{*\prime}\tilde{\phi}' + \gamma^{\alpha\beta}\partial_\alpha\tilde{\phi}^*\partial_\beta\tilde{\phi} + \right. \\ &+\; \left.\left[m^2a^2 - \Delta\xi\, a^2R + \Big((N-2)/\,2\Big)^2 K\right]\tilde{\phi}^*\tilde{\phi}\right\}. \end{aligned} \tag{39}$$

We show that the modified Hamiltonian (39) can be obtained in homogeneous isotropic space as canonical under the certain choice of variables describing scalar field. If we add N-divergence $(\partial J^i/\partial x^i)$, to the Lagrangian

density (3), where in the $(\eta, \mathbf{x})$ system of coordinates the N-vector $(J^i) = (\sqrt{\gamma}\, c\, \tilde{\phi}^*\, \tilde{\phi}\,(N-2)/2,\, 0,\, \dots\, ,0)$, the movement equations (1) are invariant under this addition. Choosing $\tilde{\phi}(x) = a^{(N-2)/2}(\eta)\phi(x)$ and $\tilde{\phi}^*(x)$, i.e., the variables in terms of which the equation (1) has the form (7), for the field's coordinates and using the Lagrangian density $L^\Delta(x) = L(x) + (\partial J^i/\partial x^i)$, we obtain that the Hamiltonian density $\tilde{\phi}'\,(\partial L^\Delta)/(\partial\tilde{\phi}') + \tilde{\phi}^{*\prime}\,(\partial L^\Delta)/(\partial\tilde{\phi}^{*\prime}) - L^\Delta(x)$ is equal to $h(x)$, from (39). This is why the Hamiltonian (39) is a canonical one for the scalar field, if $\tilde{\phi}(x)$ and $\tilde{\phi}^*(x)$ are chosen as the field's variables.

The modified Hamilton operator can be written in form (18), but in that case $Q(\eta) = 0$ and $D_\lambda(\eta) = \Omega^2(\eta)$; under its diagonalization by $\overset{*}{b}{}^{(\pm)}_J, b^{(\pm)}_J$, operators we obtain (35) with the change $p_\lambda \to \Omega$. The oscillator frequencies $\Omega(\eta)$ then coincide with the energy of corresponding particles. The initial conditions for $g_\lambda(\eta)$, corresponding to the diagonal form in the time moment η_0 with operators $\overset{*}{a}{}^{(\pm)}_J, a^{(\pm)}_J$ (17), are

$$g'_\lambda(\eta_0) = i\,\Omega(\eta_0)\,g_\lambda(\eta_0)\,, \quad |g_\lambda(\eta_0)| = 1/\sqrt{\Omega(\eta_0)}\,. \tag{40}$$

They coincide with the initial conditions used in [7] if $\arg g_\lambda(\eta_0) = 0$ is fixed. In the case of radiation dominated background ($R = 0$) they coincides with conditions used in [6, 16].

We show that the density of the particles corresponding to the diagonal form of $H^{\,mod}$ and created in the nonstationary metric is finite. For this, we find the asymptotic behavior of the functions $|\beta_\lambda(\eta)|^2$ as $\lambda \to \infty$. The functions $\beta_\lambda(\eta)$ and $\alpha_\lambda(\eta)$ that are the solutions of (32) and satisfy identity (31) can be represented as

$$\beta_\lambda(\eta) = \frac{i}{2}\frac{e^{i\,\Theta(\eta_0,\eta)}}{\sqrt{\Omega}}\left(g'(\eta) - i\,\Omega\,g(\eta)\right), \tag{41}$$

$$\alpha_\lambda(\eta) = \frac{i}{2}\frac{e^{i\,\Theta(\eta_0,\eta)}}{\sqrt{\Omega}}\left(g^{*\prime}(\eta) - i\,\Omega\,g^*(\eta)\right), \tag{42}$$

where $\Theta(\eta_1, \eta_2) = \int\limits_{\eta_1}^{\eta_2} \Omega(\eta)\, d\eta$. In consequence of (41), (42) and equation (8) the functions $s_\lambda(\eta) = |\beta_\lambda(\eta)|^2$ and $f_\lambda(\eta) = 2\,\alpha_\lambda(\eta)\,\beta_\lambda(\eta)\exp[-2i\,\Theta(\eta_0,\eta)]$ satisfy the system of equations:

$$\left\{\begin{aligned} & s'_\lambda(\eta) = \frac{\Omega'}{2\,\Omega}\,\mathrm{Re} f_\lambda(\eta)\,, \\ & f'_\lambda(\eta) + 2\,i\,\Omega\,f_\lambda(\eta) = \frac{\Omega'}{\Omega}\,(1 + 2s_\lambda(\eta))\,. \end{aligned}\right. \tag{43}$$

Taking into account the initial condition $s_\lambda(\eta_0) = f_\lambda(\eta_0) = 0$ (as $\beta_\lambda(\eta_0) = 0$) we write the system of differential equations (43) in the equivalent form of the system of Volterra integral equations

$$f_\lambda(\eta) = \int_{\eta_0}^{\eta} w(\eta_1)\,(1 + 2s_\lambda(\eta_1))\,\exp[-2\,i\,\Theta(\eta_1,\eta)]\,d\eta_1\,, \tag{44}$$

$$s_\lambda(\eta) = \frac{1}{2}\int_{\eta_0}^{\eta} d\eta_1\, w(\eta_1)\int_{\eta_0}^{\eta_1} d\eta_2\, w(\eta_2)\,(1+2s_\lambda(\eta_2))\cos[2\,\Theta(\eta_2,\eta_1)]\,, \qquad (45)$$

where $w(\eta) = \Omega'(\eta)/\,\Omega(\eta)$. To find the asymptotic behavior of $s_\lambda(\eta)$, we restrict our consideration to the first iteration of integral equation (45) and,taking into account that $\Theta(\eta_2,\eta_1) \to \lambda(\eta_1 - \eta_2)$ as $\lambda \to \infty$, represent (45) as

$$s_\lambda(\eta) \approx \frac{1}{4}\left|\int_{\eta_0}^{\eta} w(\eta_1)\,\exp(2\,i\,\lambda\,\eta_1)\,d\eta_1\right|^2. \qquad (46)$$

Consequently, we have $s_\lambda \sim \lambda^{-6}$, and the integral in (37) is therefore convergent. Thus in this case the density of created particles is finite for 4-dimensional space-time. In the case of finite volume space ($K = +1$) the total number of created particles is finite also, the Bogolyubov transformations realized by Hilbert-Schmidt operators, and the representations of commutation relations for operators $\overset{*}{b}{}^{(\pm)}_J(\eta)\,,b^{(\pm)}_J(\eta)$ are unitarily equivalent for all time.

In the presented work the metrical, canonical and introduced modified Hamilton operators are investigated. It is shown that the density of particles created in nonstationary homogeneous isotropic space metric is finite only in the case of modified Hamiltonian (39) and the energies of such particles are equal to the oscillator frequency.

The modified energy-momentum tensor (38), introduced above, coincides with the metrical one for a conformal scalar field. In homogeneous isotropic space T^{mod}_{ik} results in the modified Hamiltonian (39) which can be obtained as well as canonical under the special choice of field's variables.

It can be seen that considering a line combination of metrical (5) and canonical (4) tensors we can certainly obtain the modified tensor (38) if the quasi-particles' energy coincides with oscillator frequency. It should be stressed that the metrical energy-momentum tensor can not be changed to T^{mod}_{ik} in the right-hand sides of Einstein's equations because T^{mod}_{ik} is not covariant conservation. However under the corpuscular interpretation of the nonconformal scalar field and when the diagonalization procedure is used, the modified Hamilton operator constructed by T^{mod}_{ik} is preferable in comparison with the metrical Hamilton operator.

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